Seamless R and C++ Integration with Rcpp: Part 3 – Detailed Example

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November 16, 2013

Outline

1. Simulation

Where does our computation spend its time?

Profiling is a tool which may tell us

Having used Rcpp and the C++ compiler to speed up[1]

- loops,
- function calls,
- element access,
- algebraic expressions
- and more,

what remains as a (sometimes considerable, but often also marginal) bottleneck?

[1] List is neither exhaustive nor sorted but for illustration.

Where does out computation spend its time?

Unless of course you are *doing real work* . . .

The random number generator (RNG).

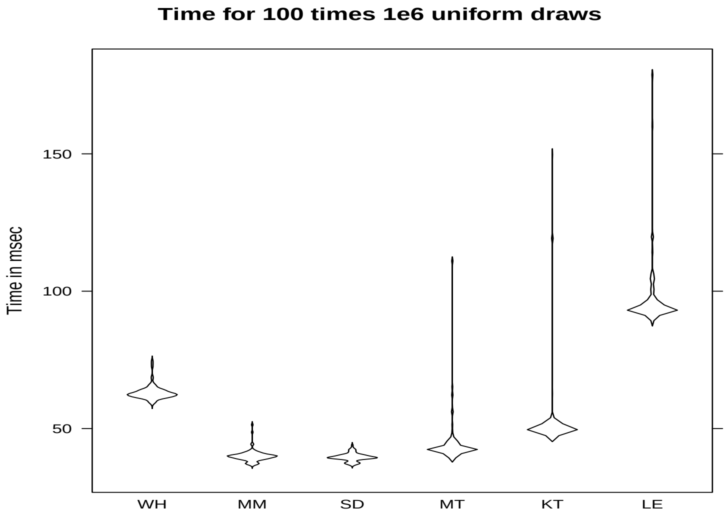
R provides two whole sets, one each for

- **uniform** Wichmann-Hill, Marsaglia-Multicarry, Super-Duper, Mersenne-Twister (the default), Knuth-TAOCP-2002, Knuth-TAOCP, L'Ecuyer-CMRG and user-supplied; and
- **normal** Kinderman-Ramage (and "Buggy Kinderman-Ramage"), Ahrens-Dieter, Box-Muller, Inversion (the default), and user-supplied

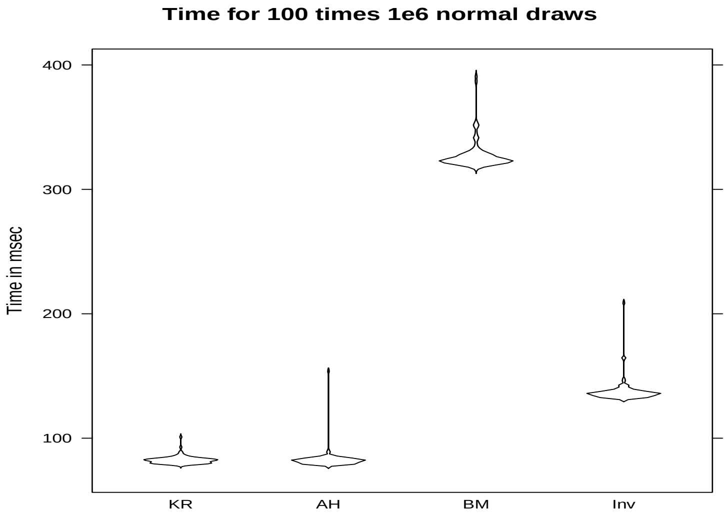
How do they perform?

RNG Speed varies

Uniform RNGs



RNG Speed varies
Normal RNGs



Outline

2 Ziggurat

The Ziggurat Generator

Marsaglia and Tsang, JSS, 2000

```
#include <math.h>
static unsigned long jz,jsr=123456789;

#define SHR3 (jz=jsr, jsr^=(jsr<<13), jsr^=(jsr>>17), jsr^=(jsr<<5),jz+jsr)
#define UNI (.5 + (signed) SHR3*.2328306e-9)
#define IUNI SHR3

static long hz;
static unsigned long iz, kn[128], ke[256];
static float wn[128],fn[128], we[256],fe[256];

#define RNOR (hz=SHR3, iz=hz&127, (fabs(hz)<kn[iz])? hz*wn[iz] : nfix())

/* nfix() generates variates from the residue when rejection in RNOR occurs. */
float nfix(void) { /* ... */ }

/* This procedure sets the seed and creates the tables */
void zigset(unsigned long jsrseed) { /* ... */ }
```

Can you spot the generator?

Can you spot another issue?

The Ziggurat Generator: Correction

Leong, Zhang, Lee, Luk and Villasenor, JSS, 2005

Using SHR3 in Ziggurat has issues and too short a cycle, use KISS (also by Marsaglia) instead.

```
#define MWC  ((znew<<16)+wnew )
#define SHR3 (jz=jsr, jsr^=(jsr<<13), jsr^=(jsr>>17), \
              jsr^=(jsr<<5),jz+jsr)
#define CONG (jcong=69069*jcong+1234567)
#define KISS ((MWC^CONG)+SHR3)

#define RNOR (hz=KISS, iz=hz&127, \
             (fabs(hz)<kn[iz]) ? hz*wn[iz] : nfix())
```

Fixes one important issue, but another problem remains.

Ziggurat widely used

But (mostly) absent from R

Ziggurat is used in a number of environments that care about speed (eg Julia, Matlab) as well as other open source environments (GNU GSL, GNU Gretl, QuantLib).

It has been reviewed well: Survey by Thomas et al (2007):

> *[W]hen maintaining extremely high statistical quality is the first priority, and subject to that constraint, speed is also desired, the Ziggurat method will often be the most appropriate choice.*

So why no R implementation (apart from a 32-bit only implementation also lacking the Leong et al correction) ?

Outline

A Simple (Virtual) Base Class

Derived classes then implement the virtual functions

```
#include <cmath>
#include <stdint.h>        // not cstdint as it needs C++11

namespace Ziggurat {

    class Zigg {
    public:
        virtual ~Zigg() {};
        virtual void setSeed(const uint32_t s) = 0;
        // no getSeed() as GSL has none
        virtual double norm() = 0;
    };
}
```

A Simple Implementation

Updated and portable version of post-2005 Ziggurat

```
#include <Zigg.h>

namespace Ziggurat {
namespace Ziggurat {
    class Ziggurat : public Zigg {
    public:
        Ziggurat(uint32_t seed=123456789) : /* ... */ {
            init();
            setSeed(seed);
        }
        ~Ziggurat() {};
        void setSeed(const uint32_t s) {/* ... */}
        uint32_t getSeed() { return jsr; }
        inline double norm() { return RNOR; }
    private:
        void init() { /* ... */ }
        inline float nfix(void) { /* ... */ }
        /* ... */
    };
}
}
```

A Simple GSL Variant

Implemented by Voss (2005), uses Mersenne Twister for uniforms

```
#include <Zigg.h>
namespace Ziggurat {
namespace GSL {
    class ZigguratGSL : public Zigg {
    public:
        ZigguratGSL(uint32_t seed=12345678) {
            gsl_rng_env_setup() ;
            r = gsl_rng_alloc (gsl_rng_default);
            gsl_rng_set(r, seed);
        }
        ~ZigguratGSL() { gsl_rng_free(r); }
        double norm() {
            return gsl_ran_gaussian_ziggurat(r, 1.0);
        }
        void setSeed(const uint32_t seed) {
            gsl_rng_set(r, seed);
        }
    private:
        gsl_rng *r;
    };
}
}
```

A Ziggurat Generator Factory

Based on generator name seed, we get a Ziggurat instance

Now we can instantiate based on user-supplied id:

```
Zigg* getZiggurat(const std::string generator, const int seed) {
    if (generator=="MT") {
        return new ZigguratMT(seed);
    } else if (generator=="LZLLV") {
        return new ZigguratLZLLV(seed);
    } else if (generator=="Ziggurat") {
        return new Ziggurat(seed);
    } else if (generator=="GSL") {
        return new ZigguratGSL(seed);
    } else if (generator=="QL") {
        return new ZigguratQL(seed);
    } else if (generator=="Gretl") {
        return new ZigguratGretl(seed);
    }

    Rcpp::Rcout << "Unrecognised generator: "
                << generator << std::endl;
    return NULL;
}
```

Outline

4. R Usage
   - Rcpp
   - R access
   - Speed

Using these functions is easy

As Rcpp Attributes makes it a breeze

```
#include <Rcpp.h>

#include <ZigguratMT.h>

// Version 1 -- Derived from Marsaglia and Tsang, JSS, 2000
static Ziggurat::MT::ZigguratMT ziggmt;

// Marsaglia and Tsang (JSS,2000)
// [[Rcpp::export]]
Rcpp::NumericVector zrnormMT(int n) {
    Rcpp::NumericVector x(n);
    for (int i=0; i<n; i++) {
        x[i] = ziggmt.norm();
    }
    return x;
}
// [[Rcpp::export]]
void zsetseedMT(int s) {
    ziggmt.setSeed(s);
}
```

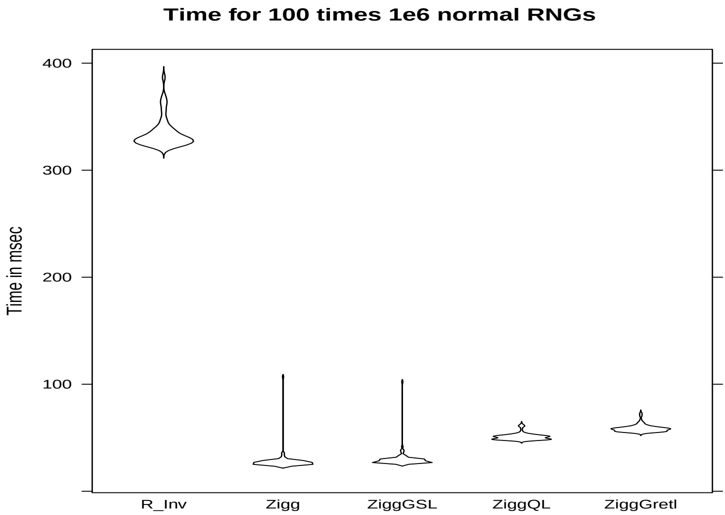
R usage

As Rcpp Attributes makes it a breeze

```r
zsetseedMT(123456)
zrnormMT(5)

## [1]  0.079374 -1.231947 -0.007092 -0.332614 -1.024207

zrnormMT(5)

## [1]  0.6725  0.3251  1.2661 -0.6573  0.2544

zsetseedMT(123456)
zrnormMT(5)

## [1]  0.079374 -1.231947 -0.007092 -0.332614 -1.024207
```

Ziggurat Speed

R versus different Ziggurat implementations



Ziggurat Speed - Table

R versus different Ziggurat implementations

```
##           test replications elapsed relative
## 2     zrnorm(N)          100   2.573    1.000
## 3  zrnormGSL(N)          100   2.863    1.113
## 4   zrnormQL(N)          100   4.964    1.929
## 5   zrnormGl(N)          100   5.667    2.202
## 1      rnorm(N)          100  33.382   12.974
```

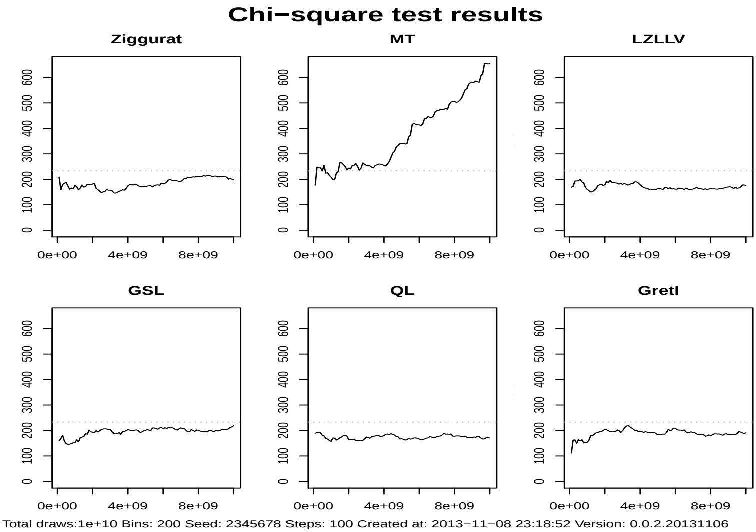
A ten to twelve-fold gain without trying all that hard ...

Outline

5 Tests
- Chi-squared (cf Leong et al, JSS, 2005)
- Standard Test
- Normal Test

Count draws in evenly spaced grid

Standard chi-square test given normal dist. cdf – 5% crit. value below

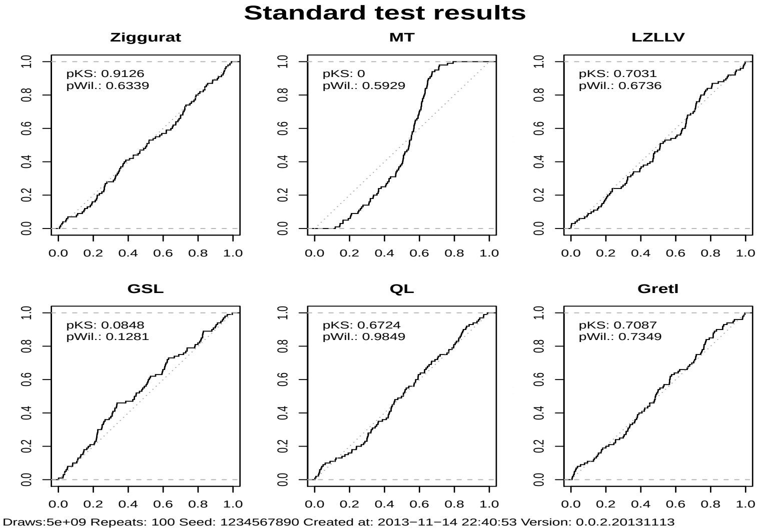


Basic Design for Standard RNG Test

Cf DieHarder by Brown, Eddelbuettel and Bauer

- Given *uniform* generator, take n draws from a $U(0,1)$. Compute sum of these n values. Repeat this m times.
- With n large enough, m results converge towards a $N(n/2, \sqrt{n/12})$ (ie. the Irwin-Hall dist. of sum of uniforms).
- Given the asymptotics, we can construct p_i for each of the m values using the inverse of Normal using the known mean and standard deviation.
- We now have m uniformly distributed values.
- Use a standard Kolmogorov-Smirnow or Wilcoxon test against departures from the uniform.

Standard Test
Based on 5e10 draws



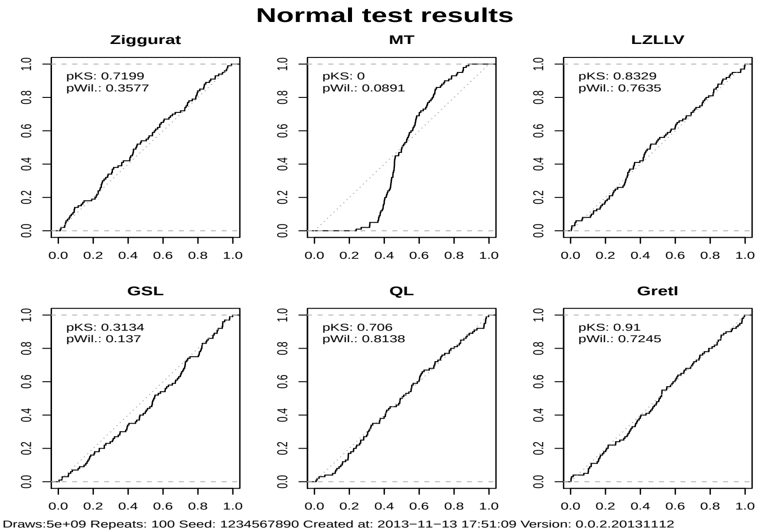
Basic Design for Normal RNG Test

Suggesting simple extension of Standard Test

- Take n draws from $N(0, 1)$ distribution. Compute sum of these n values. Repeat this M times.
- With n large enough, the m results converge towards a $N(0, \sqrt{N})$.
- Given this result, we can construct p_i for each of the m values using the inverse of the Normal using the known mean and standard deviation.
- We now have m uniformly distributed values.
- Use a standard Kolmogorov-Smirnow or Wilcoxon test against departures from the uniform.

Normal Test

Based on 5e10 draws



Outline

6 Conclusion
- Summary
- Next Steps

Ziggurat Revisited

- Updated Ziggurat: portable code for 32 + 64 bit OSs
- Clean C++ wrapper facilitates comparison & testing
- Reconfirmed Leong et al of MT failing at 32 bit limit
- Applied standard random number test (for uniforms); confirming result
- Suggested new test based directly on normal random deviates; also confirms MT issue
- Tested + validated three other OS implementations
- Timings favourable: our variant fastest across other Ziggurat implementations and the R generators

TODOs and Extensions

Things to consider

- Use (R)DieHarder and Big Crush (TestU01) tests?
- R integration – straightforward via given interface
- Look into other $U(0, 1)$ generators, too bad R does not let us access its Mersenne Twister (deliberate, see Writing R Extensions)
- Look into more Open Source variants to test?
- Look at Doornik's paper & code (not Open Source)
- Update package, complete vignette
- See the Github repo and CRAN versions

Outline

7 Rcpp Resources

Rcpp Resources

Site	`http://www.rcpp.org`
Book	`http://www.rcpp.org/book`
Gallery	`http://gallery.rcpp.org`
Code	`http://github.org/RcppCore/Rcpp`
Blog	`http://dirk.eddelbuettel.com/blog/code/rcpp`
List	`http://lists.r-forge.r-project.org/mailman/listinfo/rcpp-devel`