RcppZiggurat: Faster Random Normal Draws

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Outline

R and RNGs

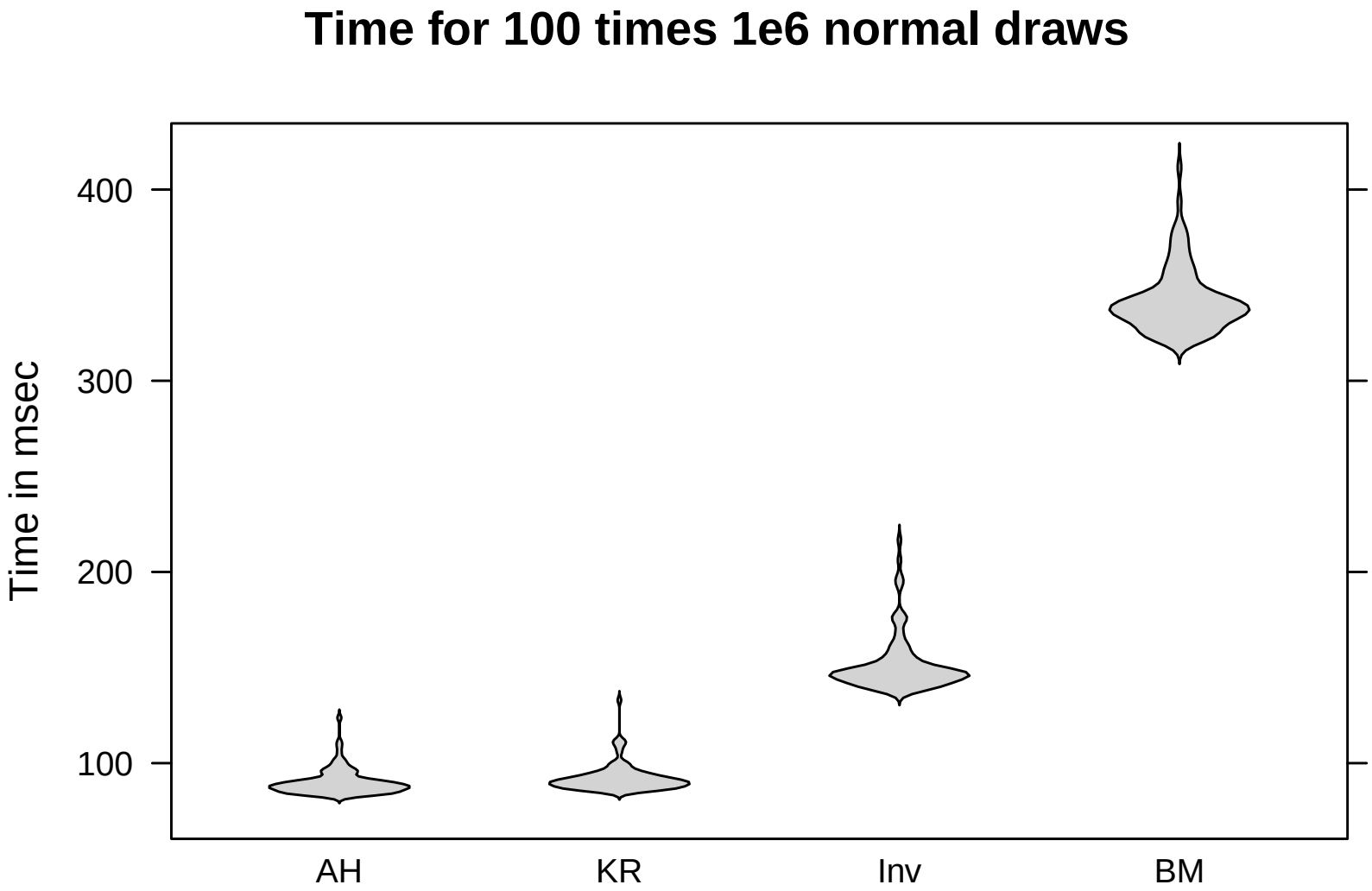
R has “batteries included”:

- Excellent support for random number generation
- Documentation isn’t all that easy to find: `RNGkind`
- There are four Normal RNGs in R (and six Uniforms)

R Normal RNGs

```
library(microbenchmark)
res <- microbenchmark({RNGkind(,"Kinderman-Ramage"); rnorm(1e6)},
                      {RNGkind(,"Ahrens-Dieter"); rnorm(1e6)},
                      {RNGkind(,"Box-Muller"); rnorm(1e6)},
                      {RNGkind(,"Inversion"); rnorm(1e6)},
                      times=100)
```

R Normal RNGs



Marsaglia and Tsang, JSS, 2000

- Introduce a very small, very fast generator
- Just a few lines of C code (in MACROS)
- Plus two support functions
- But 32-bit only (which was ok at the time...)

Marsaglia and Tsang, JSS, 2000

```
#include <math.h>
static unsigned long jz,jsr=123456789;

#define SHR3 (jz=jsr, jsr^=(jsr<<13), jsr^=(jsr>>17), \
              jsr^=(jsr<<5),jz+jsr)
#define UNI (.5 + (signed) SHR3*.2328306e-9)
#define IUNI SHR3

static long hz;
static unsigned long iz, kn[128], ke[256];
static float wn[128],fn[128], we[256],fe[256];

#define RNOR (hz=SHR3, iz=hz&127, \
             (fabs(hz)<kn[iz])? hz*wn[iz] : nfix())
```

Leong, Zhang et al, JSS, 2005

- Three page note extending Ziggurat
- Show a flaw in the underlying U(0,1) generator SHR3
- Plugging in KISS, another Marsaglia U(0,1), as fix
- This correction is not always used (!!)
- And is still 32-bit only

Widely used Open Source generators

- *Lots* of other scripting languages have Ziggurat
- It is sometimes their default
- GNU GSL has a widely used implementation by Voss
- GNU Gretl borrows from it, as does QuantLib

Outline

Virtual Base Class

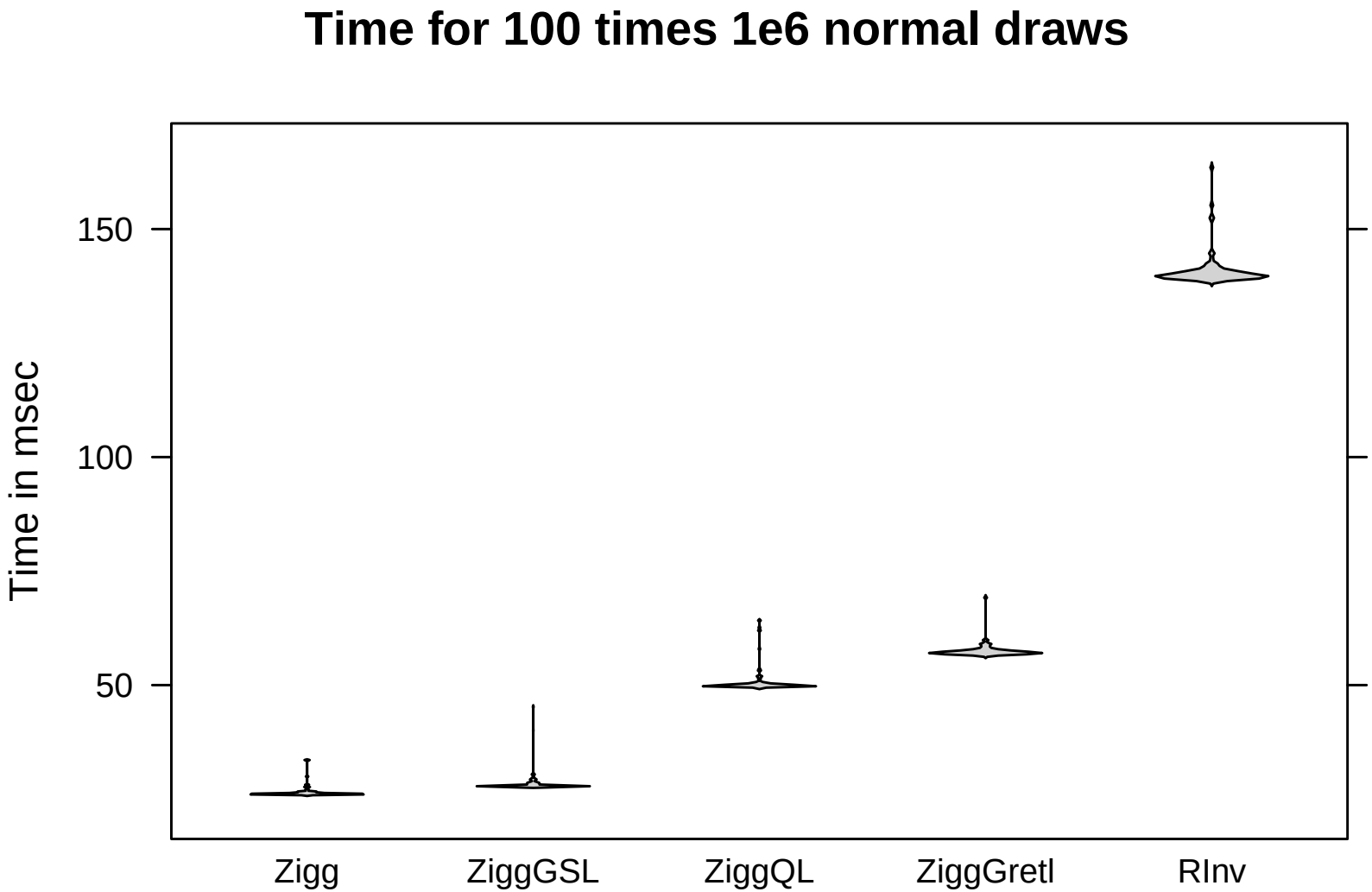
```
#include <cmath>
#include <stdint.h>   // cstdint needs C++11

namespace Ziggurat {

  class Zigg {
  public:
    virtual ~Zigg() {};
    virtual void setSeed(const uint32_t s)=0;
    // no getSeed() as GSL has none
    virtual double norm() = 0;
  };
}
```

- Our package provides several implementations
- Our default `Ziggurat` class implements LZLV in 32 and 64 bit mode
- For comparison, we also have implementations
  - ZigguratMT of Marsaglia and Tsang
  - ZigguratLZLV of Leong, Zhang et al (also the default)
  - ZigguratGSL using the GNU GSL (by linking to GSL)
  - ZigguratGl using the GNU Gretl generator (as an adapted copy)
  - ZigguratQL using the QuantLib generator (as an adapted copy)

Ziggurat RNGs

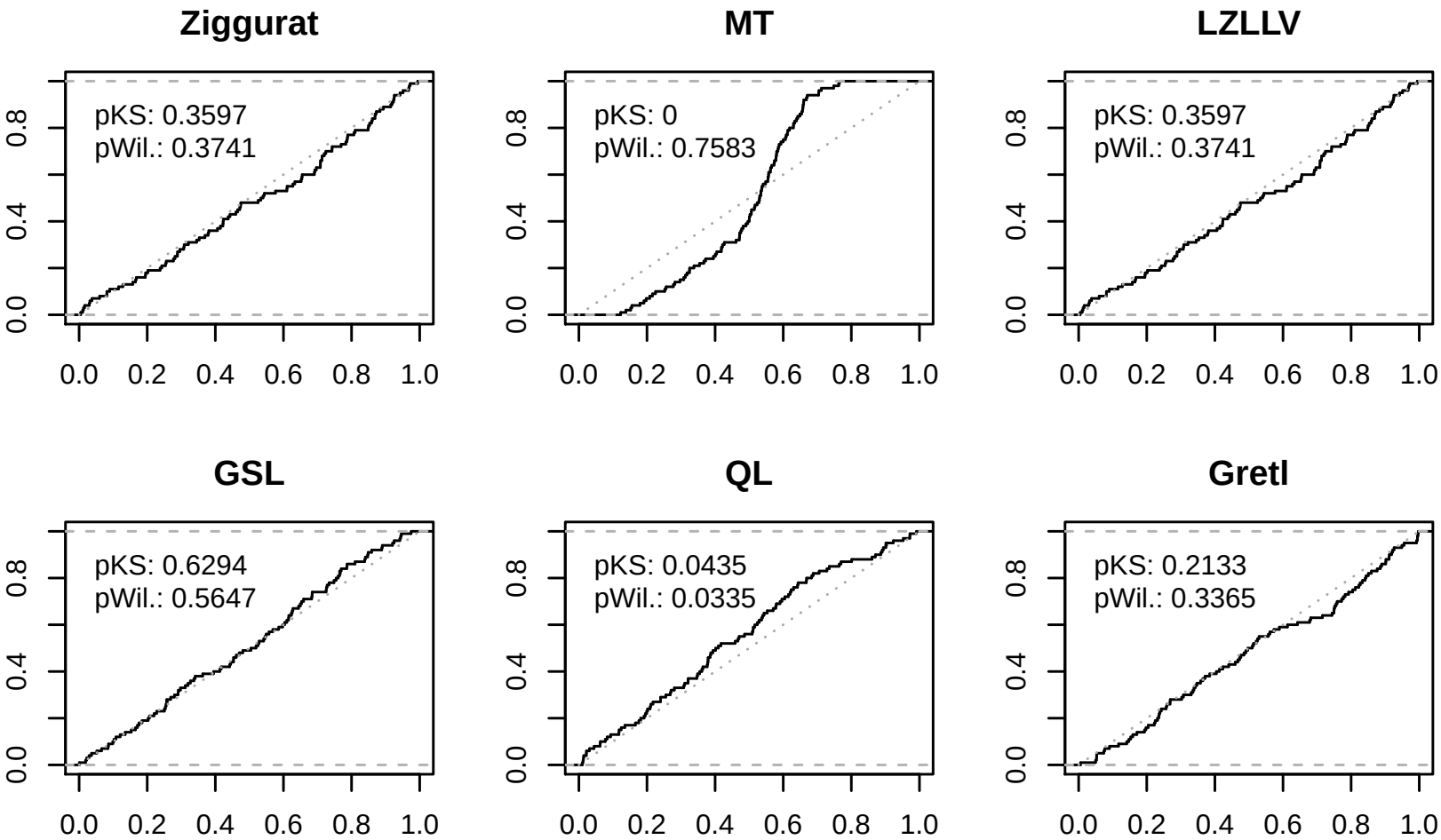


Outline

Use standard test for any Uniform generator

- *Invert each Ziggurat draw from N(0,1) to U(0,1)*
- n draws from a $U(0,1)$; compute sum of the n values.
- Repeat m times to create m sums of uniform RNGs.
- With n large, the m results converge towards $N(n/2, \sqrt{n/12})$ (Irwin-Hall distribution of sum of uniformly distributed values).
- Construct a p-value p_i for each of m values using the inverse of the Normal using the known mean and standard dev. from the Irwin-Hall distribution.
- With m uniformly distributed values: standard tests such as Kolmogorov-Smirnow or Wilcoxon can be used to test for departures from uniform.

Standard test results

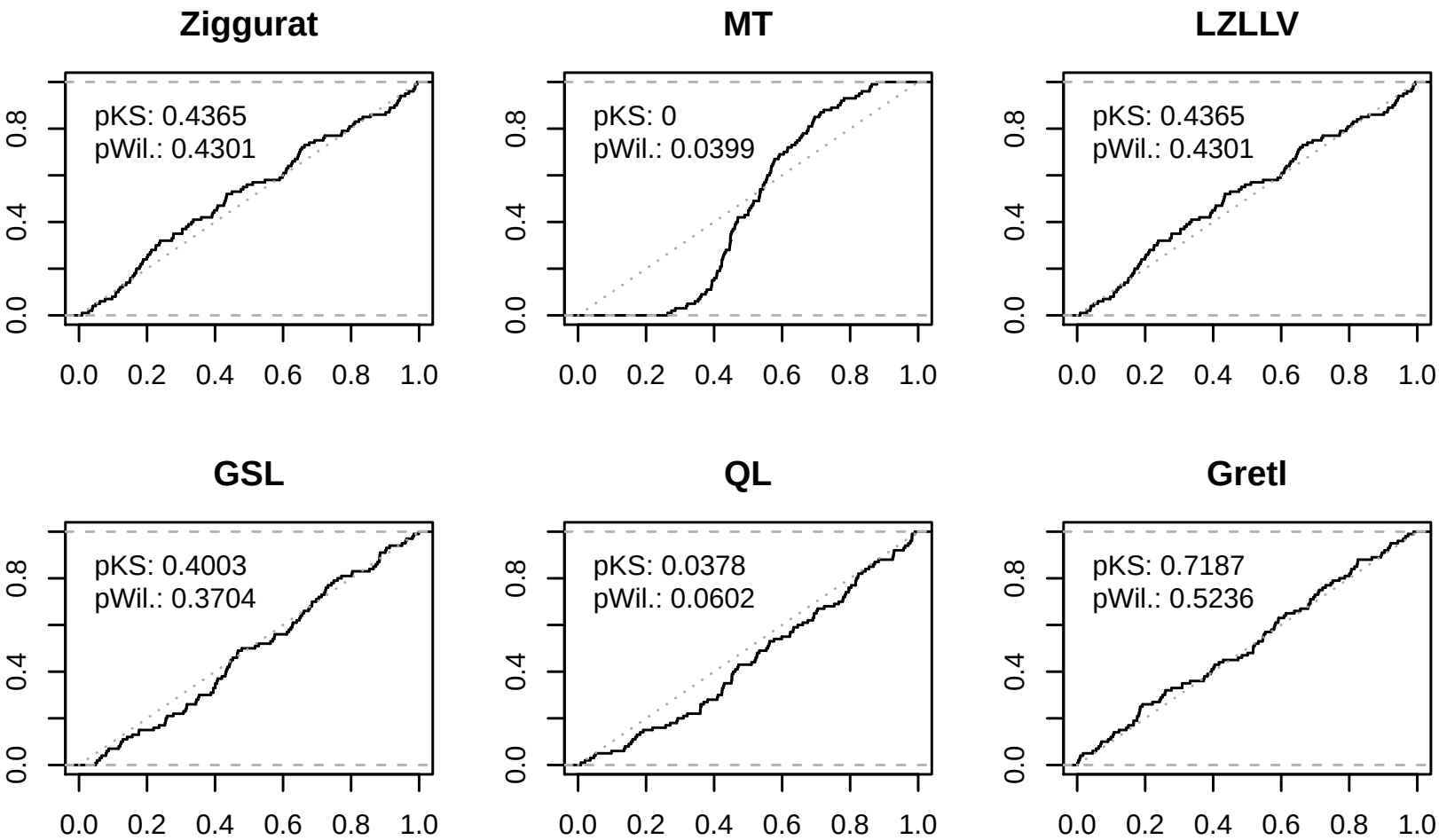


Draws:5e+09 Repeats: 100 Seed: 123456789 Created at: 2014−06−18 18:59:41 Version: 0.1.2.2

Suggested new test for N(0,1) generator

- n from a $N(0, 1)$, then compute sum of the n values.
- Repeats m times to create m sums of N(0,1) draws.
- With n large, the m results converge to $N(0, \sqrt{n})$.
- Construct a p-value p_i for each of the m values using the inverse of the Normal distribution.
- With m uniformly distributed values: standard tests such as Kolmogorov-Smirnow or Wilcoxon can be used to test for departures from uniform.

Normal test results

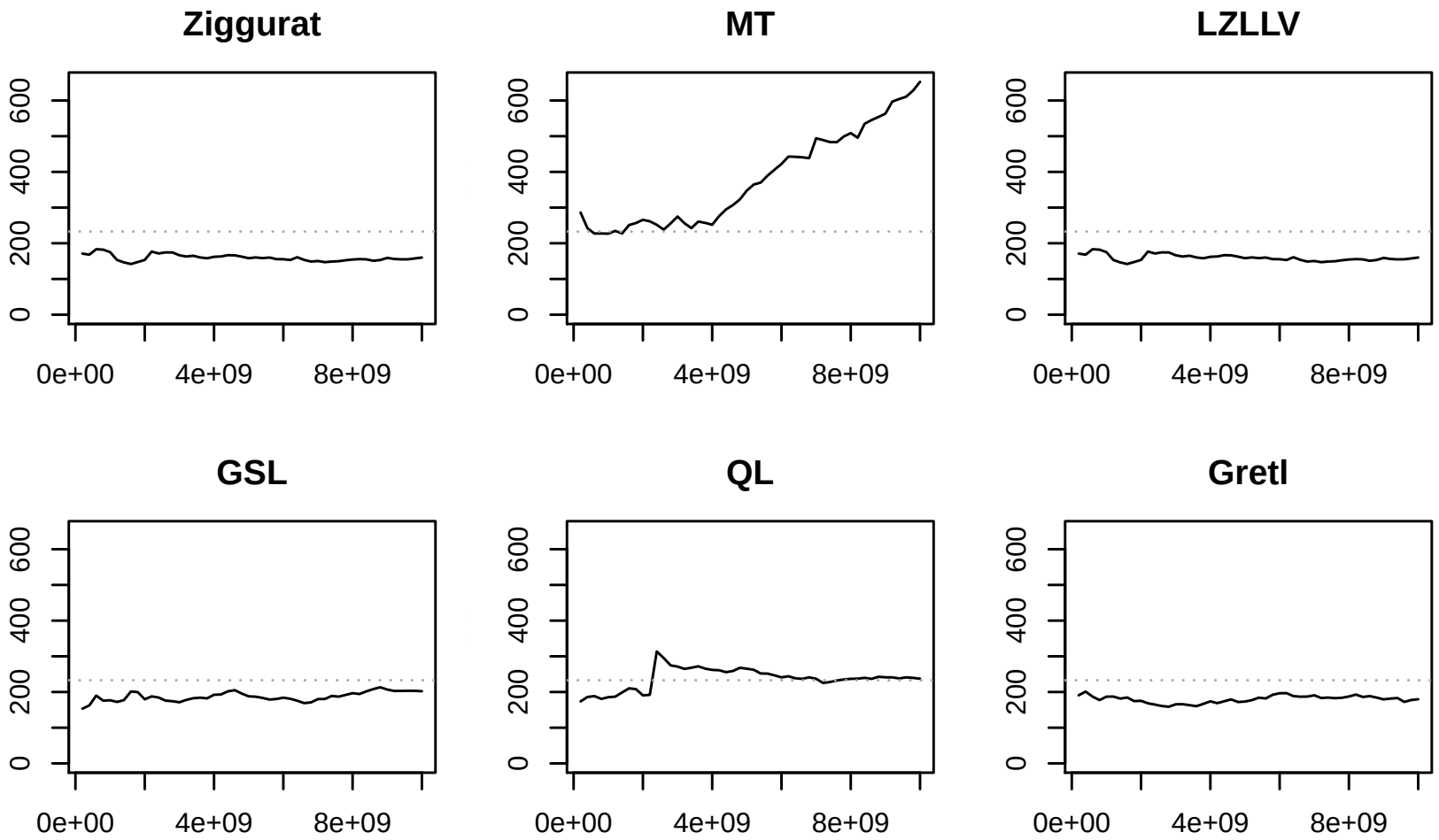


Draws:5e+09 Repeats: 100 Seed: 123456789 Created at: 2014−06−18 10:24:03 Version: 0.1.2.2

Chi-square test by Leong et al

- Divide real line into B equally spaced bins such that no $N(0,1)$ draw should exceed furthest.
- Follow Leong et al: range from -7 to 7 with 200 bins.
- Large number of $N(0,1)$ drawn; for each of a counter in the bin corresponding to the draw is increased.
- After N draws, the empirical distribution is compared to the theoretical (provided by the corresponding value of the Normal density function) using a standard chi^2 test.

Chi−square test results



Total draws:1e+10 Bins: 200 Seed: 123456789 Steps: 50 Created at: 2014−06−18 07:03:45 Version: 0.1.2.2

Outline

Comparison to R, Boost and C++11

We posted an Rcpp Gallery article 'Timing normal RNGs' comparing the Normal RNG generators of R (via Rcpp), Boost and C++.

We can now include the (default) Ziggurat method:

```
R> print(res[,1:4])
             test replications elapsed relative
4       zrnorm(n)          500   1.397    1.000
3 cxx11Normals(n)          500   4.917    3.520
2 boostNormals(n)          500   5.114    3.661
1  rcppNormals(n)          500   6.166    4.414
R>
```

Outline

RcppZiggurat

- RcppZiggurat implements updated Ziggurat generators which now work for 32- and 64-bit OSs
- Simple use: Include a single header (per generator)
- Generator is fast, small, tested and has minimal state
- This makes it particularly useful for thread-local use in parallel simulations.
- Package has been on CRAN for a few months, will get update in the next few weeks.